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## INDEX NUMBERS AND THE STANDARD OF VALUE.

II.

In the preceding paper a slight sketch was given of the development of the index number up to and including the work of Jevons on this subject. It is not intended to prolong this sketch. The literature of the index number, after Jevons, must be studied at first-hand, although it is hardly an exaggeration to say that almost every writer since Jevons -- Laspeyres, Drobisch, Lehr, Pierson, Oker, and Padan among them—has begun with Jevons's riddles and finished by condemning his standard because of its obvious conflict with the requirements of the consumption standard. Even Professor Edgeworth, who was the first to explain the merits of Jevons's work in this line, began by criticising it adversely.<sup>1</sup> Thus, the modern discussion of the index number not only began with Jevons, but in the present paper—which is devoted to an exposition of what may be called, relatively speaking, the "best standard of value"—it will be seen that the hints and suggestions embodied in Jevons's work have not yet been thoroughly exhausted.

It will be remembered by those familiar with Jevons's work on this subject that he arbitrarily altered some of the price variations which he considered excessive. The specific offense—to which he himself calls attention—consisted in toning down certain variations in the prices of cotton, hemp, flax, and tallow, which had been abnormally raised by the effect of war in the United States and Russia. Jevons's action cannot be considered as intrinsically wrong or unfair. Neither can it be unequivocally indorsed. Everything, of course, depends upon what stage in the proceedings we are to give ourselves over into the hands of the science of probability. If, as Jevons intimated was the

<sup>\*</sup> See Journal of the Statistical Society, December, 1883.

case, we are to assume ignorance as soon as the price variations have been recorded, and are to take all variations on an equal footing, then he was guilty of breaking the prearranged rules of the game. If, on the other hand, we possess the criteria by which to adjudge one set of returns inadmissible, why not use our knowledge to extend the process of elimination until only the correct variation (or set of variations) remain? To be able to decide that one set of returns does not measure the change in the value of money, implies the possession of at least a vague notion of what constitutes the true standard of value. Why not clarify this notion and extend the field of actual knowledge instead of summoning to our aid the weapons of ignorance, powerful though they be? Just as in every physical measurement the limitations of human sensibility and the imperfection of mechanical instruments necessitate the employment of averages and the logic of probability, so in the measurement of value we will always be compelled to use these devices at some point in the proceedings. But this does not relieve us of the necessity of excluding every observation or return that is known to be erroneous.

The principle of exclusion employed by Jevons, however, can scarcely be said to furnish the criterion by which to decide whether a given commodity may be included in the measurement or not. He modified the price of cotton because his object was to ascertain the change in the value of money due to the increase in the precious metals, and the extraordinary variation in the price of cotton was confessedly due to other causes. "Our duty," he wrote, "as regards fluctuations due to changes of supply of commodities is to have nothing to do with them, but to eliminate the effects from our inquiry as soon and as completely as possible." "

Such an explanation is about as mysteriously irritating as his riddles respecting the proper mean to be employed in averaging price variations. Nevertheless, it rests upon a broad basis of truth, and finds acceptance not only in economic theory,

<sup>&</sup>lt;sup>1</sup> A Serious Fall, etc., pp. 25, 26.

but in the common sense of the business man. The latter would be quite unwilling to include in the standard of value a commodity whose supply was twice as great in T' as in T'', yet this is what he often does when he accepts an ordinary index number as an expression of the change in the value of money. The classical theory of money, to refer to the science itself, incorporated the same idea in the axiom "value depends upon demand and supply." If the supply were visibly and confessedly different in the two epochs there would be good grounds for the belief that the value of the commodity had changed.

But the truth, towards which Jevons was groping, was a more important one than that incorporated in the negative statement that a commodity whose supply has changed in a large degree cannot be included in a standard of value. What we want to know is what constitutes a constancy of value. The reply to that question was given later by Jevons himself, and in clearer and more precise terms by those writers who have developed the theory of marginal utility which he propounded.2 Value is a function of quantity consumed: the "ultimate" or "best" standard of value is a restricted consumption standard, each commodity of which is consumed in the same quantity in T' as in T''. In anticipation of the objections of those readers who admit that marginal utility is a function of quantity, but deny that the same is true of value, it may be stated that reasons will be given hereafter to show that in the measurement of changes in the value of money, at least, no real distinction exists between value and utility or marginal utility.

Since the appearance of the Austrian theory of value a number of monographs have been published in which the standard of value is discussed from the standpoint of the marginal utility theory of value, and at least two utility standards have been suggested which require notice here. An examination of their

<sup>&</sup>lt;sup>1</sup> Theory of Political Economy, p. 165 et passim.

<sup>&</sup>lt;sup>2</sup> See MARSHALL, Principles, Appendix, Note XII; and IRVING FISHER, Mathematical Investigations in the Theory of Value and Prices, pp. 18, 87, 89.

rationale will demonstrate how perfectly natural, even commonplace, a standard of utility is.

The merit of first proposing a standard of utility adapted to the measurement of changes in the value of money must, so far as I am aware, be ascribed to Dr. Julius Lehr.\* Dr. Lehr describes the fundamental theory upon which the whole group of utility measurements rest, as follows:

Um nun die Aenderungen des Geldwerthes bemessen zu Konnen, mussen auf einem bestimmten Gebiete (Land, Provinz) Alle Waaren und Leistungen, für welche Preise gezahlt worden sind, berücksichtigt werden. Hierauf sind dieselben auf ein gemeinschaftliches Masz zu bringen. Eine Handhabe hierfür bietet die jeweilige Gleichwerthigkeit. Man Kann nämlich diejenigen Mengen von Waaren und Leistungen einander gleich setzen, welche als gleich werthig zu betrachten sind. Als gleichwertig aber haben wir in unserem Falle, in welchem es sich nur um die Begriffe Preis, Durchschnittspreis, Marktpreis handelt, die jenigen Mengen anzusehen, für welche gleich viel gezahlt wird. Ist der Preis eines Hektoliter Wein=60 Mark, der eines Festmeter Buchenscheitholz=10 Mark, so sind 6 Festmeter Holz einem Hektoliter Wein gleich zu setzen. Für eine Mark erhalten wir dann 10 Hektoliter Wein, ebenso auch 10 Festmeter Holz. Diese Mengen wollen wir als je eine "Genusseinheit" bezeichnen, ein Begriff, der in Folgender Weise zur Berechnung der Veränderungen des Geldwerthes benutzt werden kann.2

Expressed in our notation Dr. Lehr would say that at any given time  $\frac{1}{p_1}c_1$  or  $\frac{1}{p_2}c_2$  - - - or  $\frac{1}{p_n}c_n$  each yields or is equal to a unit of utility; or expressing the same idea in another way: utility of  $c_1$ : utility of  $c_2$ : utility of  $c_n$ ::  $p_1$ :  $p_2$ :  $p_n$ . But at this point we are met by the difficulty that the prices change. At any given time the utility of various commodities is proportional to their prices, but what is to be done when these prices change?

Dr. Lehr met this difficulty by computing a formula for the average price of a commodity. If in T',  $q_1'$  units of  $c_1$  are sold at  $p_1'$  per unit, while in T'',  $q_1''$  units of  $c_1$  are sold at  $p_1''$ , per unit, then the total price of  $(q_1'+q_1'')$  units of C is  $(q_1'p_1'+q_1''p_1'')$ , and on the average of T' and T'' one unit of  $c_1$  costs  $q_1' p_1'' + q_1'' p_1''$ . This amount, Dr. Lehr declares, is the true average of T' and T'' one unit of T' on

<sup>&</sup>lt;sup>1</sup> Beiträge zur Statistik der Preise (Frankfort, 1885).

<sup>&</sup>lt;sup>2</sup> Ibid., pp. 37, 38.

age price of  $c_1$  because this price multiplied by the quantity of  $c_1$  actually bought in T' and T'' will give the amount actually spent for  $c_1$  in T' and T''. In consequence:

ut. of  $c_1$ : ut. of  $c_2$ : ut. of  $c_n$ ::

$$\frac{q_{1}^{'} p_{1}^{'} + q_{1}^{''} p_{1}^{''}}{q_{1}^{'} + q_{1}^{''}} : \frac{q_{2}^{'} p_{2}^{'} + q_{2}^{''} p_{2}^{''}}{q_{2}^{'} + q_{2}^{''}} : \frac{q_{n}^{'} p_{n}^{'} + q_{n}^{''} p_{n}^{''}}{q_{n}^{'} + q_{n}^{''}}.$$

If in T'  $(q_1'' p_1'' - \cdots + q_n'' p_n'')$  dollars buy  $(q_1'' c_1 - \cdots + q_n'' c_n)$ , and in T''  $(q_1'' p_1'' - \cdots + q_n'' p_n'')$  dollars buy  $(q_1'' c_1 - \cdots + q_n'' c_n)$ , then  $\frac{V''}{V'} = \frac{q_1'' c_1 - \cdots + q_n'' c_n}{q_2' c_2 - \cdots + q_n'' c_n} \times \frac{q_1' p_1'' - \cdots + q_n'' p_n'}{q_1'' p_2'' - \cdots + q_n'' p_n''}.$ 

Substituting for  $c_1$ , - - -  $c_n$  their respective values as determined by their average prices, we get Dr. Lehr's formula:

$$\frac{V''}{V'} = \frac{\left(\frac{q_1' \ p_1' + q_1'' \ p_1''}{q_1' + q_1''}\right) q_1'' - - - + \left(\frac{q_n' \ p_n' + q_n'' \ p_n''}{q_n' + q_n''}\right) q_n''}{\left(\frac{q_1' \ p_1' + q_1'' \ p_1''}{q_1' + q_1''}\right) q_1' - - - + \left(\frac{q_n' \ p_n' + q_n'' \ p_n''}{q_n' + q_n''}\right) q_n''} \times \frac{q_1' \ p_1'' - - - - + q_n' \ p_n''}{q_1'' \ p_1'' - - - - + q_n'' \ p_n''}.$$

The validity of Dr. Lehr's method depends upon the average he employed. If his formula is proposed as a mere empirical approximation, it may be credited with that degree of validity appertaining to a mean between two erroneous results, whose respective errors are not known to be of an opposite kind. From a theoretical standpoint there is no superior virtue—in fact, no virtue at all—in the average he used.

According to Dr. Lehr, commodities have an importance proportional to their prices. If the price of  $c_1$  in T' is  $p_1'$ ,  $\frac{1}{p_1'}$ 

<sup>1</sup> Beiträge, p. 39. By similar reasoning we might say that the average quantity of  $c_1$  consumed in T' and T'' is  $\frac{q_1' \not p_1' + q_1'' \not p_1''}{p_1' + p_1''}$ . Weighting the prices with these average quantities we get the following consumers' index-number, given as (5) on page 7 of the preceding article (JOURNAL OF POLITICAL ECONOMY, December, 1901).

$$\frac{V''}{V'} = \frac{\binom{q_1' \ p_1' + q_1'' \ p_1'}{p_1' + p_1''} p_1' - - - + \binom{q_n' \ p_n' + q_n'' \ p_n'}{p_n' + p_n''} p_n'}{\binom{q_1' \ p_1' + q_1'' \ p_1''}{p_1' + p_1''} p_1'' - - - + \binom{q_n' \ p_n' + q_n'' \ p_n''}{p_n' + p_n''} p_n''} p_n''}.$$

units of  $c_1$  contains one unit of utility,  $c_1$  contains p' units of utility, and its importance may be represented by p'. Similarly, the importance of  $c_1$  in T'' may be represented by  $p_1''$ ; but  $p_1'$ and  $p_1$ " cannot be added, since they are incommensurable. We know that the importance of  $c_1$  in T' is  $p_1'$ , because its money value is  $p_1'$ . We also know that the importance of  $c_1$  in T'' is  $p_1$ ", because its price is  $p_1$ ". But unless the value of money has remained the same  $p_1'$  and  $p_1''$  are expressed in different units. In other words, Dr. Lehr has assumed the value of money to be the same in the two epochs, in order to measure the change in the value of money by another process. Dr. Lehr seems to have been led into error by forgetting that prices represent relations or ratios between amounts of utility and not absolute amounts of utility. In T' the utility of  $c_1$  may be represented by  $p_1'$ , and in T'' the utility of  $c_1$  may be represented by  $p_1''$ , but we know nothing of the ratio  $\frac{\text{ut. of } c_r \text{ in } T'}{\text{ut. of } c_r \text{ in } T''}$ . The "genusseinheit" of T' is not the "genusseinheit" of T''. Prices represent ratios between amounts of utility, not positive amounts of utility.

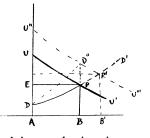
Lehr's formula is valid in two special cases, in both of which our problem might easily be solved without the unwieldy formula which he has given. It is evident that the process is correct, where ut. of  $c_1$  in T' =ut. of  $c_1$  in T''; ut. of  $c_n$  in T' =ut. of  $c_n$  in T''. The formula will also be correct where  $\frac{p_1'}{p_1''} = \frac{p_2'}{p_2''} = \frac{p_n'}{p_n''}$ . When the utility of commodities remains unchanged from T' to T'', or when the variation has been the same in all prices, Lehr's formula holds good. The only instance in which either of these phenomena can occur is where the variation in prices lies wholly on the side of money. Lehr's method agrees with all its predecessors in being applicable only in that one special instance, which never occurs.

What we have called the weighted labor standard has been again proposed as a measure of marginal utility by Professor Clark.<sup>1</sup> If units of labor-time be measured along *AB*, and the

<sup>&#</sup>x27;Yale Review, November, 1892.

ordinates for the corresponding time points represent the utility and disutility resulting from the corresponding unit of labor, we get the familiar utility and disutility curves UU' and DD', respectively, which intersect at the point P. It is evident that the value or marginal utility of the day's product—ABPE—is

equal in amount to the marginal disutility of the day's product, both being equal to the number of units of labor time consumed when BP is taken as the unit of utility and disutility. Hence Professor Clark's conclusion that at any given time, and in respect to any isolated laborer the value of his product is most easily measured by the amount of labor time expended in producing it.



In modern life no one produces for his own consumption. We obtain the goods we wish to consume by producing others which we exchange for them. A produces  $c_1$ , B produces  $c_2$ , C produces  $c_3$ .  $c_1$  is distributed among A, B, and C in return for their work, or the products of their labor. The value of  $c_1$  is not measured by the time it takes A to produce it, but by the time it takes A, B, and C to produce the goods which are exchanged for it. The same argument applies to every product of the society; the value of each commodity is best measured

by the social or aggregate labor time spent in acquiring it. "The standard for measuring it (value) is the sacrifice in final

periods of labor, entailed on society in acquiring it." 2

But value, Professor Clark holds, is a social phenomenon.<sup>1</sup>

Professor Clark's standard is evidently the old labor standard made up of all kinds of labor in the relative proportions in which they are expended. The defects of this standard have already been considered. In the first place labor is like other commodities in respect to the fact that at different times the various kinds of labor are purchased in different amounts. In the second place, as is shown by the dotted lines in the last dia-

See New Englander, Vol. IV (n. s.), p. 457.

<sup>&</sup>lt;sup>2</sup> Yale Review, November, 1892, p. 272.

gram, the unit of disutility, or the length of the day's work, must, ex hypothesei, vary when the productivity of labor changes. Thus, as is shown in the diagram, if the utility curve changes from UU' to U'''U''', we will have a new unit of disutility (and of marginal utility), B'P'. The relation of BP to B'P' is unknown—is, in fact, the ratio we seek. However, what is more likely to happen in such a case, is a change in the relation of utility and disutility. The standard of life will change, the laborer will demand more utility per unit of disutility; he will perhaps continue to work AB hours, but will demand for the marginal unit of disutility a larger amount of utility, BD'', than before.

So obvious are these facts that I question whether Professor Clark proposes his standard as one suitable for the actual measurement of variations in value from time to time. From the present standpoint, however, he demonstrates nothing more than the proposition that at any given time a certain quantity of labor may be used as an ultimate standard of value—a proposition that has been repeated hundreds of times in the phrase "any commodity will serve as a measure of value at a given time and place."

There is, consequently, little assistance to be derived from the formulæ of Professor Clark and Dr. Lehr. The one is a weighted labor standard, the other a new variety of the compromise consumption standards noted on page 7 of the preceding article. Both have been shown to be theoretically defective, and both require statistical data which it is almost impossible to secure.

The utility standard proposed in this paper, however, requires only a few commodities, the *per capita* consumption of which is approximately the same in T' and T'', and manifestly rests upon the proposition that the value or utility of a commodity is constant as the quantity consumed is constant. An apparent objection to this proposition is found in the fact that the utility of a commodity is a function, not only of its own consumption quantity, but in some cases is also a function of the quantities of other commodities. Thus, if an individual consume X units of

tea and Y units of coffee in T', while in T'' he consumes X units of tea and NY units of coffee, it is possible that the utility of tea would not be exactly the same in T' as in T''.

In the above illustration it is evident that the error caused by regarding tea as invariable in T' and T'' would be insignificant. That part of the value of tea which is dependent upon the quantity of coffee consumed is, under ordinary circumstances, a quantity of the second order which may be safely neglected. But in certain other commodities this interdependence of values cannot be neglected. Some goods are practically interchangeable. "When herring are dear the people buy sprats." They may not like sprats so well, but both satisfy hunger.

In consequence, it is not wholly true that a quantitative change in the consumption of a commodity is a certain indication of a change in its utility. On the other hand, it is more nearly true that fixity in quantity indicates fixity of utility, because when the quantity of one "complementary" or "substitutionary" good changes, the quantity of the other also changes. In all such instances a theoretical remedy lies in the exclusion of both commodities from the standard, when the quantity of either changes. The practical remedy, however, lies in the fact that the utility of a great majority of commodities is so little dependent upon the quantity of other commodities that this factor is negligible.

In another and more vital sense the value of any commodity may be said to be a function of the quantity of other goods. Excluding substitutes, complementary and competing goods it is evident that the utility of any commodity depends upon the quantity of other commodities, because of the mobility of labor and capital. If, through invention or discovery, labor and capital become more productive in one industry, while the conditions of production in other industries experience no variations of their own, labor and capital will flow from the first to the other industries. The effect of the invention or discovery will thus be spread over the whole industrial mechanism, increasing the output in all lines and decreasing the utility of every commodity.

But changes in the conditions of production are not all of one kind. Opposed to the march of invention is the law of decreasing returns. Each progressive step in any industry operates to send some of its labor into other industries, thus increasing the supply of all commodities. Every retrogressive step has an opposite effect. Two general forces are thus continually being propagated. If these forces are equal as well as opposite, they will neutralize each other, leaving those industries which have experienced no proper variations unaffected. If one of these forces be greater, producing a resultant general force, we may expect to see this resultant neutralized in those industries which have experienced an equal and opposite variation of their own. Such industries will furnish the standard commodities desired. It is not necessary that the quantity of such goods should be the same throughout the whole interval T'-T''. It is only necessary that the quantities be the same in T' as in T''. Neither is it essential that the standard should always consist of the same commodities. In comparing T' and T'' we may use the commodities  $c_1$ ,  $c_3$ ,  $c_5$ ; while in comparing T'' and T'''', the commodities  $c_2$ ,  $c_1$ ,  $c_4$  may be used.

Looking now to the consumption of the individual, we find a similar interdependence of consumption quantities. When the price of the commodity  $c_1$  rises, the quantity consumed immediately falls. But this will not, in most cases, be the only effect; the quantities of other commodities will also fall. If  $c_1$  be some "indispensable" commodity—a medicinal article, for instance —the fall in the quantity of  $c_1$  may be small, while the quantities of other goods—e. g., certain luxuries—may decrease in a greater degree.  $c_1$  having become more valuable, other goods must become more valuable. In accordance with a principle which may be called the mobility of consumption, the utility of a good is a function of the quantities of all other goods. As in the preceding case, variations are not all of one kind between T' and T''. Positive price variations set in action general forces which are neutralized in certain lines of consumption by the general forces propagated by negative price variations.

In such lines of consumption we find the standard commodities desired. It is interesting to note that the mobility of consumption is practically instantaneous in its operation. The flow of labor and capital from one industry to another is obstructed in many ways. But no similar obstructions are found in expending money for consumption.

But production and consumption are, of course, interdependent. Whether or not a good is consumed in the same quantities in T' and T'' depends upon the interaction of two forces: the change in the price of the good, and the change in the income of society. Prices may rise uniformly. In this case, if incomes rise in the same ratio, commodities may be consumed in the same quantities in T' and T''. What actually happens, however, is that some prices rise and others fall. Whether the standard commodities will come from the group whose prices rise or the group whose prices fall depends upon the movement of income between T' and T''. If incomes have risen in T'' the society will be enabled to consume the usual quantities of the goods whose prices have increased. If incomes have fallen, only those goods whose prices have fallen will continue to be consumed in the old quantities. The forces which influence the value of money are various, but their resultant is measured in the prices of those goods which are consumed in the same quantities.

The proposed standard, then, is, without aiming to be so, a logical compromise between the labor and consumption standards. Böhm-Bawerk has mentioned this attribute as a prime requisite of the ultimate standard of value, and its importance is manifest. The consumption and labor standards, and their respective defendants, are not wholly wrong; the standards are based upon important half-truths, and the defendant of each feels in his cause a measure of justice. The trouble is that the defendant of the consumption standard never recognizes the semi-justice of his opponent's position, and *vice versa*. Men like David A. Wells assume the validity of the labor standard,

<sup>&</sup>lt;sup>1</sup> Annals of the American Academy Vol. V., Pt. 1, p. 208.

construct strong and convincing arguments from this standpoint, and then wonder why they make no impression upon the vast number of people who have no idea of, or at least no sympathy with, any other than the consumption or tabular standard. The reason lies in the simple fact that the value of money depends upon both the level of prices and the size of the income. Whether or not money was more valuable in 1896 than in 1873 depends upon whether the effect of the fall in prices was great enough to offset the effect of the increase in wages.

The standard of value proposed here rests upon the marginal utility theory of value, and, although described simply as a standard of value, is in its first interpretation a standard of utility. The nature of the unit or standard of utility has been the subject of considerable speculation.<sup>2</sup> Some writers seem to have assumed that it would necessarily be a unit of sensation. Others have raised unfortunate distinctions between standards of marginal and total or absolute utility. Almost all of them have assumed that the standard of utility was hopelessly impracticable. An examination of the more important of these objections will serve to show that the standard of utility is not only not impracticable, but that it really constitutes the only practicable standard by which changes in the value of money can be measured.

In the discussion of the proper standard of deferred payments, carried on several years ago between Professor Ross and Dr. Merriam,<sup>3</sup> the impression was created that, since value is measured by marginal utility the correct standard of value would be a standard of marginal utility as distinguished from a standard of total or absolute utility.

This distinction arises from a misconception of the nature of utility which is mischievous, not only because of its prevalence,

<sup>&</sup>lt;sup>1</sup> See Wells, Recent Economic Changes, chap. v.

<sup>&</sup>lt;sup>2</sup> Ross, Annals of the American Academy, November 1892, and November, 1893; MERRIAM, ibid., January, 1893; FETTER, ibid., May, 1895; MENGER, Revue d'Economie Politique, February, 1892.

 $<sup>^3</sup>$  Annals of the American Academy, November, 1892; January, 1893; November, 1893.

but because of the fact that a majority of persons, if called upon to choose between a standard of total utility and one of marginal utility, would without hesitation prefer the former. It is, therefore, important to show that the differentiation of marginal from total utility in the standard of value is impossible, and that, in consequence, the standard proposed here is not open to the objections formulated by Professor Ross and Dr. Fetter against the theoretical standard defended by Mr. Merriam.

In order to gain any definite conception of a value constant in different epochs, it is necessary to make value equivalent to that property of goods which has been called "utility," "desiredness," "ophelimity," "real value," etc., and for the purpose of measuring variations in value it is necessary to postulate that the amounts of this property contained by various goods are proportional to their respective prices. This fact is most frequently expressed in the simple statement that at any given time marginal utilities are proportional to prices. It is usually deduced as follows: The last dollar spent for every commodity must yield the same amount of utility, or I would buy more of some commodities and less of others. Thus, if the last dollar's worth of bread yields me less utility that the last dollar's worth of meat, I will buy more meat and less bread until the equilibrium be restored. Hence, marginal utility of  $\frac{1}{p_1}$  units of  $c_1 =$ marginal utility of  $\frac{1}{p_2}$  units of  $c_2$ . This expression is then expanded so as to read:

$$\frac{\text{mar. ut. of I unit of } c_{\text{I}}}{\text{mar. ut. of I unit of } c_{\text{2}}} = \frac{p_{\text{I}}}{p_{\text{2}}}.$$

But this expression is, by admission, only approximately true. By the law of variable utility the utility of one unit of  $c_1$  is not  $p_1$  times the utility of  $\frac{1}{p_1}c_1$ . Consequently, writers have been careful to say that this law holds only when  $c_1$  and  $c_2$  are very small, or, in other words, that the law is strictly true only for differential increments,  $dc_1$ ,  $dc_2$ , etc.

<sup>&</sup>lt;sup>1</sup> Annals, November, 1893, and May, 1895.

The law which states that marginal utilities are proportional to prices is, according to this analysis, evidently open to three criticisms which have been frequently urged against it. It is asserted: (1) that it postulates the infinite divisibility of commodities; (2) that it holds only for infinitesimal marginal increments; (3) that a fixed amount of marginal utility is an empty philosophical vagary, impractical and unsuited (even if practical) to act as a standard of value.

These criticisms have originated in a defective or, at least, an inadequate analysis of consumption. In order to test their validity it is desirable to examine more closely into the nature of the utility with which we are concerned.

In speaking of utility it is necessary to limit the connotation of the word by reference to a definite commodity, and most important of all, a definite time epoch. We must think of the utility of the commodity  $c_1$  in the epoch T. The omission of the time element in discussions of utility is largely responsible for the objections noted.

In order to look more closely into the process of consumption, a simple illustration may be employed. Suppose an individual to heat his house with a ton of coals per month. The commodity unit here will be the ton of coals; the time unit the month; and the utility unit the total utility afforded by the ton of coals. We may further assume that the ton is divided into a number of uniform subdivisions, say bushels. The following inferences may be drawn:

- I. That irrespective of the actual amounts of utility afforded by the several bushels the individual will regard the utility afforded by each bushel as the same. Either at the beginning or at the end of the month each bushel will appear equally important, though it may be noted that we are here wholly beyond the domain of exchange value.
- 2. That so far as there is any law discernible the actual process of consumption tends to follow the process of thought. Notwithstanding the gradual satiability of want, equal subdivisions of the unit of commodity tend to yield uniform amounts

of utility. Cæteris paribus, the individual will distribute the time and divide the commodity so that equal portions of the commodity will yield like amounts of utility, because in this way the maximum amount of satisfaction will be obtained. Shorten the time unit to the dinner hour, include only one class of commodities—food—think of utility as the mere power of satisfying hunger, and we obtain the concept of utility that has fathered the above criticisms and inveigled economics into the hopeless search for a unit of sensation. A definite quantity of a commodity designed to be consumed in a definite period of time will be consumed in a way such that equal subdivisions of the commodity will tend to yield uniform amounts of utility.

Utilities, then, are proportional to prices, and the relation holds good for total utility and actual commodities as well as for marginal utility and infinitesimal increments. The individual regulates his purchases with reference to a general level of possible utility, which is dependent upon the amount of his income. Knowing his total income he lays out fixed amounts for the different goods. He knows that he can purchase so much  $c_1$  so much  $c_2$ , so much  $c_3$ . The more he buys of  $c_1$ , the less satisfaction per unit will be receive, and the same is true of  $c_1$ ,  $c_2$ ,  $c_3$ . But each subdivision of  $c_1$  does not yield less satisfaction than every preceding subdivision. Each succeeding increment of  $c_1$ may bring down the amount of utility which would be rendered by preceding subdivisions, if the last increment were not consumed. But the latter statement is by no means equivalent to the former statement. Taking the annual consumption of a commodity as the unit, it is clear that we cannot treat the first increment or subdivision consumed in January as more useful, more desirable, or more valuable than the last increment consumed in December.

It follows also that the differentiation of a standard of marginal utility from the standard of total utility is impossible. We imagine a case in which a man has only one loaf of bread a week. We deduce from this the correct conclusion that in this event the utility of the loaf would be practically infinite. We

then imagine the man to have ten loaves per week and from the second case deduce the incorrect result that the utility of the first loaf is still infinitely greater than the utility of the tenth loaf. Such an inference is entirely unwarranted. It assigns to the first loaf of ten the utility which it would possess if the individual had only one loaf. The result in our problem is to furnish us a standard of potential utility with which we have no concern. There is no difference between a standard of marginal and a standard of total utility. The distinction is between utility and potential utility.

As there is no useful distinction between marginal and total utility, there would seem to be no distinction between utility and value, unless the basis of differentiation be those moral or non-economic criteria according to which we adjudge that a quart of whisky possesses less utility than a loaf of bread, or a bushel of coals more utility than a diamond. With such distinctions the theory of value has no concern. Accordingly, I see no reason to employ any other term than the simple word value. I dispense with the word utility the more willingly because many prominent writers (with a verbal justification, I must admit) deny that two physically dissimilar goods can be said to possess like amounts of utility. The utility afforded by a dollar's worth of candy and that afforded by a dollar's worth of quinine, they say, are incommensurable. Terms are not material. If the utility afforded by the two goods is not the same, the economist has nothing to do with utility. His subject-matter is that property which two commodities may be affirmed to possess in like amounts when the individual desires neither to the exclusion of the other. If one finds his pathway blocked by a post, it is useless to argue; the sensible thing is to avoid the post.

Before closing it is necessary to say a few words of the practical application and advantages of the proposed standard.

At the outset it must, of course, be admitted that the absolute requirements of this standard cannot be satisfied any more

<sup>1</sup> See Fisher, Mathematical Investigations, pp. 3-24; cf. Sherwood, "The Philosophical Basis of Economics," Publications of the American Academy, No. 209; and Marshall, Fortnightly Review, April, 1876, pp. 596, 597.

than the absolute requirements of any physical measurement can be satisfied. The reasons for this are obvious. While we can reasonably hope for the necessary statistics of consumption, as soon as the government becomes awake to the importance of such data, we do not at present possess them. On the other hand, if such statistics were available, they would seldom or never show a commodity whose per capita consumption was exactly the same in T' and T''. We should be forced to include commodities whose quantities had varied a little, say 5 per cent., in the interval; and this would introduce an element of error which could only be removed by the use of a mean. At some point in the investigation we must "trust to luck," average the returns in such a way as to make the errors mutually destructive, and test the result by the criteria of the science of probability.

But at just what point we shall begin to trust to luck is a question of the greatest practical importance. If we attempt to ascertain the mean human stature of a nation, to borrow an illustration from Dr. Venn, it is very evident that we must not only measure an immense number of people, but we must select these people judiciously. It will not do to take most of the measurements in the cities and a very few from the rural districts, or all from the East and none from the West, because this will influence the mean type which we are trying to elicit. Moreover, it is plain that this mean type is a very elusive affair. At bottom, it seems largely a creature of definition. We can modify it by confining our investigation to a limited area. It measures no known force or set of forces. It is useful chiefly for purposes of comparison; it is valuable, for instance, to know that on the average Englishmen are so many inches taller than Frenchmen.

On the other hand, to borrow another illustration from Dr. Venn,<sup>2</sup> almost verbatim, suppose that a man had been firing at a small mark on a wall, that the mark had subsequently been

<sup>&</sup>lt;sup>1</sup> Venn, *Logic of Chance*, chap. 2: "It need hardly be insisted upon that the interest and significance of such investigations as these are almost entirely dependent upon the statistics being very extensive," p. 24.

<sup>2</sup> Ibid., chap. 19, § 2.

removed, and we were asked to guess the position of the mark from the arrangement of the shot-marks. It is evident here that, unless some regular force, such as the wind, had exerted an unvarying influence upon the shots, we should be able to locate the mark with all necessary precision from a comparatively small number of shot-marks. The fact that we had been aiming at a fixed object would simplify the matter greatly. And until the position of the wafer had been estimated by the appropriate average, each shot would be an equally valid indication of the position desired.

The relation between the two problems just illustrated is very similar to the relation between the average of all price variations obtainable and the average of those commodities whose consumption quantities have not varied. The former requires an immense number of prices, which must be representative as well as independent, and when the variations of these prices have been averaged the meaning and importance of the result are uncertain. As in the first illustration, its chief utility is probably comparative; that is, it would possibly furnish a valid method of *comparing* the general movement of prices in two intervals of time, or in two countries for the same interval of time. This is a very different thing, however, from a *measurement* of change or difference in the value of money between two epochs or places.

The index number suggested here, however, partakes of the objectivity of a physical measurement, in which each of the returns is a slightly inaccurate measurement of the same thing. If the theory be correct upon which this index number is based, our result will be more exact the more exclusively we confine ourselves to those commodities which are consumed in exactly the same quantities in T' and T''. With a standard approaching this condition within, say 5 per cent., as suggested, the margin of probable errors would be small, although it might happen that one or two of the variations diverged greatly from the rest, and for this as well as other important reasons it would be advisable to use the median. But, because of the "objectivity" of the measurement and the consequent restriction of the errors,

there would not be the same necessity for a large number of prices that there is in the ordinary index number, and the accuracy of the result would not be so largely dependent upon the selection of a representative list of independent price variations.

In the ordinary consumption index number, the importance of selecting a truly representative list of independent variations is as great as the task is difficult. In the actual computation of index numbers this difficulty expresses itself in the concrete question: Ought we to take a few very important commodities whose price variations are wholly distinct, or should we include as many prices as possible, placing more emphasis upon quantity than upon importance and independence? "Common sense" furnishes no guide here, as it is extremely desirable both that the commodities should be important and the price variations numerous, and, moreover, "common sense" is a notoriously poor guide in delicate quantitative questions. The importance of the problem is shown by the fact that if we throw the index numbers of Falkner, Soetbeer, and Sauerbeck upon the same basis, the first (based upon 223 articles) will in practically every year be found higher than the second (114 articles), and the second higher than the third (45 articles). While the grave discrepancies between these index numbers is probably due in a large degree to actual differences in the variation of money in the United States, Germany, and England, they are probably due in a greater degree to the number and character of the commodities covered by the several measurements.2

In the index number proposed here, however, this question would not arise, as the importance of the commodity in consumption would have little or no weight. The change in the price of pepper is just as good an indication of the variation in the value of money as the change in the price of wheat. Neither would it be fatal to include a disproportionate number of prices from one general field of industry. If, for instance, five equally

<sup>&</sup>lt;sup>1</sup>See the interesting debate upon this question between Mr. Sauerbeck and Professor Pierson, in the fifth volume of the *Economic Journal*.

<sup>&</sup>lt;sup>2</sup> See Aldrich, Report upon Wholesale Prices, Wages, and Transportation. Part I, p. 256 et passim.

expert surveyors measure the length of a river with the same instruments, and one of the surveyors make twice as many measurements as any other, no grave error could be introduced by taking an average of the returns without reference to the particular individual who made them. Moreover, no weights would be required and our result would have a definite meaning in itself; it would be the best measurement we could make of the change in the value of money, not an ambiguous "average variation of prices."

In conclusion it may be added that the theory upon which this standard is based furnishes a test of the direction of the variation in the value of money, which may prove not without interest to those impartial investigators who have hesitated between the claims of the labor and consumption standards and who, in consequence, have been uncertain whether the value of money rose or fell between 1860 and 1891, to take the standard and final years of the Aldrich investigation, for example. test is found in the consumption of those commodities whose prices were the same in 1860 and 1891. If a dollar purchased  $c_1$ ,  $c_2$ ,  $c_n$ , etc., in both 1860 and 1891, then the value of each of these commodities must have varied exactly as the value of money varied between 1860 and 1891, and the direction of this variation will be shown by the change in the consumption of these articles. If the consumption of these articles increased, the value of money fell in the interval; if the consumption decreased, the value of money rose. A fortiori, if the consumption of those articles whose prices have risen increased, the value of money must have fallen.

Table II, Part I, of the Aldrich report on Wholesale Prices, Wages, and Transportation shows that the prices of the following articles were the same in 1860 and 1891: potatoes, milk, soda crackers, broadcloths, coal (anthracite, pea), chestnut lumber in the log, wooden tubs, Ontario starch. The following articles were from 2 to 10 per cent. higher in 1891 than in 1860: lamb, beef (loins), beef (ribs), rye flour, Sumatra pepper (whole), eggs, anthracite coal (egg and stove), sugar of lead (white),

linseed oil. To base a verdict concerning the change in the value of money upon these data, without accurate statistics of consumption, is of course out of the question, but in connection with the more extensive price lists from which they are taken and the general belief that the consumption of meat and coal, for instance, is increasing, they furnish some reason for the belief that in the last half century, to use round figures, the value of money has fallen rather than increased, or, in other words, that the increase in wages has more than offset the fall in prices.

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